# Localization regime in diffusion MRI: theory and experiments

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**Introduction:** In 1991, Stoller *et al.* predicted the emergence of the localization regime at high gradients g when the transverse magnetization is localized near boundaries of the sample and the signal E exhibits an unusual stretched-exponential decay: $-\ln(E) \propto g^{2/3}$[1]. We investigated this phenomenon for diffusion of xenon-129 gas inside cylinders and outside an array of rods.

**Methods:** PGSE experiments were conducted with hyperpolarized xenon-129 gas in 3D-printed phantoms (see [2] for details). Numerical computations of the signal and the magnetization for cylinders were performed with the matrix formalism [3].

**Results and discussion:** Figure 1(top) shows the signal attenuation inside a cylinder of diameter 2R=3.8 mm. Experimental points are in perfect agreement with the exact solution [3], while its high-gradient behavior is accurately captured by the first two eigenmodes of the Bloch-Torrey operator [4,5], from which we derive:

$$E \approx C \exp\left(-|a_1| \frac{\ell_\delta^2}{\ell_g^2} - \frac{\ell_\delta^2}{R^{1/2}\ell_g^{3/2}} + \frac{\sqrt{3}\,\ell_\delta^2}{2|a_1|R\ell_g}\right) \qquad \text{Eq. 1}$$

where $|a_1|\approx 1.02$, $\ell_g = (\gamma g/D_0)^{-1/3}$, $\ell_\delta = (D_0\delta)^{1/2}$, and C is a slowly decreasing function of g computed numerically from the eigenmodes. This stretched-exponential behavior results from localization of the magnetization near the boundary points at which the gradient is orthogonal to the boundary, see Fig. 1(bottom). Similar behavior was observed for hindered diffusion outside an array of rods [6].

**Conclusion:** We showed the emergence of the localization regime for both confined and hindered diffusion at gradients as small as 10 mT/m and $bD_0$ above 4. The high-gradient decay of the signal is reproduced by the asymptotic formula (1) containing no fitting parameters. Our results imply that this behavior is a generic feature of diffusion MRI that can be observed in most scanners. Even though the localization regime is yet poorly understood and exploiting its potential advantages is still challenging, the high sensitivity of the signal to the microstructure at high gradients is a promising avenue for creating new experimental protocols.

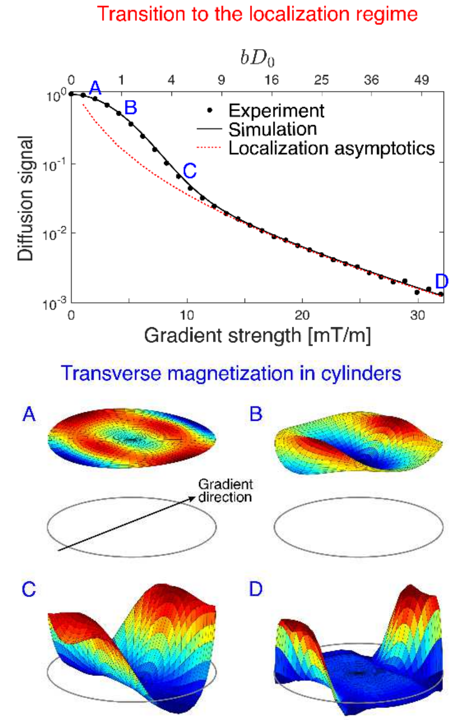


Figure 1: (Top) Signal attenuation inside a cylinder of diameter 2R=3.8mm, obtained with a PGSE sequence with pulse duration δ=6 ms, inter-pulse time Δ=9.34 ms, and gradient strength g ranging from 0 to 32 mT/m. With the estimated free diffusion coefficient $D_0\approx(3.7\pm0.2)\cdot10^{-5}$ m$^2$/s, the diffusion length $\ell_\delta = (D_0\delta)^{1/2} \simeq 0.5$ mm, while the gradient length $\ell_g = (\gamma g/D_0)^{-1/3}$ varies from 0.8 mm to 0.25 mm with g. The b-value is $b = \gamma^2 g^2 \delta^2 (\Delta - \delta/3)$. (Bottom) As the gradient increases (four plots correspond to g=2, 5, 10, and 32 mT/m), simulated magnetization is getting more localized near two boundary points.

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